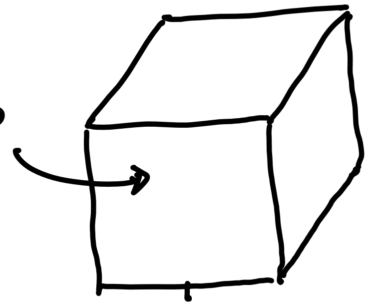
Note Title 1/23/2008

3D infinite potential well

$$\begin{cases} V = 0 & -\frac{L}{2} < x,y,z < L/2 \\ V = \infty & \text{outside} \end{cases}$$

V = 0



$$-\frac{\hbar^2}{2m}\nabla^2\psi + \cancel{V}^{\,0}\psi = E\psi$$

$$\psi = \psi_x(x)\,\psi_y(y)\,\psi_z(z) \Rightarrow$$

$$-\frac{\hbar^2}{2m}\left(\psi_x''\psi_y\psi_z + \psi_x\psi_y''\psi_z + \psi_x\psi_y\psi_z''\right) = E\psi_x\psi_y\psi_z$$

$$-\frac{\hbar^2}{2m}\left(\frac{\psi_x''}{\psi_x} + \frac{\psi_y''}{\psi_y} + \frac{\psi_z''}{\psi_z}\right) = E$$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{\psi_x''}{\psi_x} = E_x, \quad -\frac{\hbar^2}{2m}\frac{\psi_y''}{\psi_y} = E_y$$

$$-\frac{\hbar^2}{2m}\frac{\psi_z''}{\psi_z} = E_z \quad \text{and} \quad E_x + E_y + E_z = 0$$

We can relate E_x, E_y, E_z to k_x, k_y, k_z:

$$E_x = \frac{\hbar^2 k_x^2}{2m} \qquad E_y = \frac{\hbar^2 k_y^2}{2m} \qquad E_z = \frac{\hbar^2 k_z^2}{2m}$$

$$E = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2 + k_z^2\right)$$

From B.C.: $k_x = \frac{n_x \pi}{L}$, $k_y = \frac{n_y \pi}{L}$, $k_z = \frac{n_z \pi}{L}$

$$E = \frac{\hbar^2}{2m} \frac{\pi^2}{L^2} (n_x^2 + n_y^2 + n_z^2) \quad n_{x,y,z} = 1, 2, 3, \ldots$$

Ground State is:

$$E_1 = E_{111} = \frac{\hbar^2 \pi^2}{2mL^2} (1^2 + 1^2 + 1^2) = \frac{3\hbar^2 \pi^2}{2mL^2}$$

degenerate states (3 fold)

$$\underbrace{E_{211} = E_{121} = E_{112}}_{\text{degenerate states (3 fold)}} = \frac{\hbar^2 \pi^2}{2mL^2} (1+1+4) = 2E_1$$

$$\underbrace{E_{123} = E_{132} = E_{213} = E_{231} = E_{312} = E_{321}}_{\text{Six fold degenerate}} = \frac{14}{3} E_1$$

Six fold degenerate

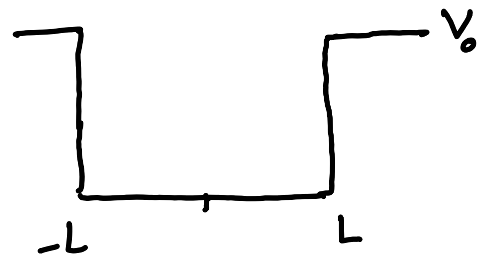
Degeneracy is a result of symmetry.

If V is changed in a way that the symmetry is broken, a new set of energy eigenvalues is generated.

Breaking the symmetry $\Rightarrow$ reduces degeneracy

Finite potential well (particle in the well)

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$



note

Since V is symmetric $V(x) = V(-x) \Rightarrow$

ψ has eithe odd or even parity

$$\psi(x) = \pm\psi(-x)$$

For $x < |L|$ we have:

$$-\frac{\hbar^2}{2m}\psi'' = E\psi \rightarrow \psi'' = -k^2\psi \qquad \left(E = \frac{\hbar^2 k^2}{2m}\right)$$

$$\rightarrow \psi = A\sin kx + B\cos kx$$

For $x > |L|$ we have:

$$-\frac{\hbar^2}{2m}\psi'' + V_0\psi = E\psi$$

$$\psi'' = \underbrace{\frac{-2m(E - V_0)}{\hbar^2}}_{K^2}\psi = K^2\psi$$

(note: $E < V_0$)

$$\rightarrow \psi = Ce^{-Kx} + De^{Kx}$$

Since ψ must be finite $\Rightarrow$

$$\psi = \begin{cases} C e^{kx} & x < -L \\ D e^{-kx} & x > L \end{cases}$$

In Summary:

$$\psi(x) = \begin{cases} C e^{kx} & x < -L \\ A \sin kx + B \cos kx & -L < x < L \\ D e^{-kx} & x > L \end{cases}$$